REVIEWS

Water Waves: The Mathematical Theory with Applications, by J. J. Stoker. New York: Interscience Publishers, 1957. 567 pp. $12.00 or 88*s*.

Most students of fluid dynamics are familiar with the chapter on water waves in Lamb's *Hydrodynamics*. Fewer, however, are aware of the many interesting developments in the theory of water waves since the last edition of Lamb's treatise; developments stimulated no less by the temporary demands of the second World War and the constant need for engineering and oceanographic knowledge than by the intrinsic mathematical interest of the subject. Professor Stoker's long-awaited book is the first serious attempt, since the war, to give a mathematical account of this rapidly growing field.

Two things about the book stand out immediately. First it is written almost purely from the mathematical point of view. There is little of the quantitative comparison with observation so desirable in all branches of fluid dynamics, and not least in the study of water waves.

Secondly, only certain parts of the field are covered; for example, there is no account of surface tension or viscosity, of internal waves, or of standing waves of finite amplitude. At the same time, some very specialized topics, such as the wave pattern produced by a ship steering an exactly circular course, have been treated in detail.

The book is divided into four parts. The first part, consisting of two short chapters, is devoted to a derivation of the general equations of irrotational motion for a non-viscous fluid, and to some well-known theorems required later in the book. The peculiar difficulty arising from the non-linear boundary conditions at the free surface is noted. Two basic approximations are described, namely the 'small-amplitude' approximation of Airy and Stokes, where the solution is developed in powers of the ratio of wave height to wavelength; and the 'long-wave' or 'shallow-water' approximation of Boussinesq and Rayleigh, which is more appropriate when the ratio of mean depth to wavelength becomes small.

Part II of the book then describes some solutions with the small-amplitude theory (to a first approximation only) and Part III describes solutions with the shallow-water theory. Finally, Part IV contains two somewhat dissimilar problems, namely the problem of the breaking dam and the question of the convergence of the small-amplitude method of approximation. These are loosely associated as "problems in which the free surface conditions are satisfied exactly".

In Part II a conventional treatment is first given of simple harmonic waves in water of uniform depth, with an interpretation of group-velocity both as the velocity of propagation of a wavelike disturbance (represented in the form of a sum or integral) and as the velocity of propagation of energy. Chapter 4 treats the motion due to a harmonic distribution of pressure over part of a free surface. One of the principal mathematical tools used in this section of the book is complex variable theory, combined with suitable radiation conditions at infinity. In chapter 5 the small-amplitude theory of waves on sloping beaches is described using the artificial assumption of a logarithmic singularity at the shoreline. In reality, of course, the energy is dissipated in some other way, either in friction at the bottom or by breaking of the waves, so that non-linear terms would certainly be important. However, the linear problem has been elegantly solved by Peters and Roseau for arbitrary angles of inclination of the beach. Professor Stoker gives also his own method of solution for special angles $\pi/2n$. Similar complex-variable methods are applied to the problem of waves breaking against a flat ' dock ' and against a vertical ' cliff '. In the latter case the solution is hardly realistic, since no account is taken of the reflected energy. The section includes a solution of Sommerfeld's problem of the diffraction of waves round a vertical wedge.

The second subdivision of Part II, entitled " Unsteady motions ", treats transient motions started from rest, using a Fourier transform technique combined with Kelvin's stationary phase approximation. Similar techniques are used in the third subdivision, dealing with waves on running streams (or waves caused by a moving pressure pulse). In the latter problem the question arises how to determine the uniqueness of the solution; for a free wave of arbitrary amplitude may always be added to any solution of the free surface conditions. One way is to assume a small but finite frictional dissipation of energy. Professor Stoker makes the more satisfactory assumption that the steady state is the result of starting the motion from rest and waiting until the transient motion has died down. According to this assumption, the transient energy is radiated away to infinity.

The three-dimensional pattern caused by a moving pressure pulse (Kelvin's ship-wave problem) is treated in chapter 8, in some detail, although with no discussion of the ' cusp-line ' where the wave form is expressible in terms of Airy functions. Part II ends with a chapter giving a linearized theory of ship motion, in which it is also assumed that the ratio of the ship's thickness to length is small. Though similar in some respects to Haskind's theory, it ignores terms that would lead to a damping of the pitching, surging and heaving motions, and so cannot account for these phenomena. As in all the preceeding chapters, no quantitative comparison with observation is given.

In Part III we come to the author's discussion of waves in shallow water. Chapter 10, which is largely a reproduction of a paper by the author (*Comm. Pure Appl. Math.* **1**, 1948, 1), is the longest in the book and is the one whose validity seems most open to question. The theoretical background is roughly as follows.

In water of finite depth, the convergence of the small-amplitude method depends upon the parameter

$$\epsilon = ak/(kh)^3$$

being small compared with unity, where a denotes the wave amplitude,

h the mean depth and k is the inverse of some typical horizontal dimension such as the wavelength. When the depth diminishes, so that ϵ is of order unity or greater, another method of approximation (the shallow-water approximation) may be used; this involves expanding the solution in powers of $(kh)^2$.

It is easily shown that in the first approximation to the shallow-water theory all disturbances are propagated without change of form, with velocity $(gh)^{1/2}$. In the second approximation any progressive wave has in general a small second-order deformation; if we set this deformation equal to zero we obtain a non-linear equation for the first approximation, which yields a class of steady progressive waves, the 'cnoidal' waves of Korteweg & de Vries (1895). As the amplitude diminishes these tend to the well known sine-waves of Stokes, and as the wavelength is increased one obtains, in the limit, the solitary wave of Rayleigh and Boussinesq. Convergence both for the solitary wave and for cnoidal waves has recently been proved by Friedrichs & Hyers (1954) and by Littman (1957). The work of Korteweg & de Vries showed further how a shallow-water wave of arbitrary form will gradually change shape, becoming sometimes steeper in front and sometimes less steep according to circumstances.

Professor Stoker makes only a cursory reference to the work of Korteweg & de Vries, and takes no advantage of their excellent elucidation of the nature of long waves. His account of these matters, particularly of the limitations inherent in the shallow-water theory, is too diffuse and leaves much in doubt. In the method of treatment with which most of chapter 10 is concerned, all terms of order $(kh)^2$ are entirely neglected, giving rise to what may be called a *zero-order* theory. The equations are indeed convenient to handle, being analogous to those governing the propagation of plane sound waves of finite amplitude; solutions can be constructed using the method of characteristic curves. However, this approach leads to the conclusion, for example, that any wave advancing into an undisturbed region must become steeper and eventually break, and that no steady wave can exist (apart from the trivial case of an unperturbed stream). This is directly contradicted by the existence of solitary and cnoidal waves which are well established both by theory and observation.

The zero-order theory may of course have a limited region of applicability, and a careful discussion of this point is much to be desired. But, since no account is taken of vertical accelerations, the theory can hardly apply to breaking waves; Professor Stoker's attempts to press the theory this far must therefore be viewed with reserve. His theoretical profiles of breaking waves are indeed unlike what is observed, as can be seen from Professor Munk's photographs shown in the book.

Chapter 11, entitled "Mathematical Hydraulics", is unlike the rest of the book in that mathematical elegance is sacrificed in an attempt to predict a concrete phenomenon, namely floods in the Ohio and Mississippi Rivers. An empirical law of turbulent friction is assumed, and good agreement is obtained with observation. This chapter follows closely two reports to the U.S. Army Corps of Engineers by the author, in conjunction with Isaacson and Troesch.

The final chapter of the book contains the two loosely associated problems mentioned earlier. The first of these, the breaking of a dam, is solved by the author in Lagrangian coordinates, using a series expansion in powers of the time. In fact only the terms as far as t^2 are evaluated. The problem appears to be similar to others (for example a collapsing mass of water with a sharp crest), in which difficulties are encountered in the terms in t^4, and one would be happier to see the approximation carried out to terms of higher order.

This chapter also contains a discussion of the existence of progressive waves of finite amplitude, and another version of Levi-Civita's proof for the convergence of the small-amplitude approximation. I feel that undue emphasis is given to this existence proof while the significance of other work relating to the problem is neglected. In Levi-Civita's work the ratio of wave-height to wavelength is limited to about 1/200 (see Hunt 1953) whereas experimentally much steeper waves are known to exist. The surface profile of the ' highest wave ' (that is to say, a wave having a sharp crest with 120° angle) was calculated long ago by Michell (1893). Further, Havelock (1918) extended Michell's method so as to compute the form of a wave of any amplitude less than the highest; he showed also by numerical examples that these waves were the same as those calculated by Stokes's small-amplitude approximation; and that the value of Stokes's parameter for the highest wave was approximately 0·2919. This is very probably the actual radius of convergence. From a practical point of view, Havelock's work is of no less value than Levi-Civita's and deserves at least some mention. Similarly some mention should have been made of the theoretical work of Penney & Price (1952) on standing waves of finite amplitude, and of Taylor's (1953) experimental verification that the highest standing wave has a crest angle in the neighbourhood of 90° (as compared with 120° for the progressive wave).

Since the book deals only with irrotational waves, the limitations of the irrotational theory should be pointed out. For some years now it has been known that waves are not irrotational beyond the linear (small-amplitude) approximation. No matter how small the viscosity, some vorticity always spreads inward from the boundaries and results in a mass-transport distribution quite different from that indicated by potential theory. Thus the irrotational theory is applicable only for a limited time after the wave motion is set up.

The general impression left by this book is that it is primarily a collection of personal contributions, some of more interest and validity than others, bound loosely into one volume. The scope of the author's researches is wide, but the claim to have presented " a connected account of the theory of wave motion in liquids, together with its applications " can hardly be said to be substantiated when so many important topics are omitted, and others are treated incompletely or by passing references. In particular, the central position of the cnoidal wave theory in providing the link between the small-amplitude approximation and the shallow-water approximation should have been more clearly brought out. By its very lack of unity the book may prove to be valuable as a collection of scattered work, although this is not quite what the title leads a reader to expect.

A more serious defect is the general lack of connection between the mathematics and the physics. This is particularly important in a field of applied mathematics where approximations of all kinds play an essential part. In at least one case—in the section on breaking waves—the author has been led to wrong conclusions. True, there are a number of illustrations, but with the exception of chapter 11 these are not generally presented in such a way as to make quantitative comparisons possible.

The mathematical text is well reproduced, but the photographs and diagrams are of poor quality in a few cases. In particular figures 6.6.1, 10.9.1, 10.10.22 and 12.2.1 need redrawing, being rather crude sketches which could easily have been made more precise by use of available data. Many better illustrations of a solitary wave are available than figure 10.9.2.

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Proceedings of the 5th Midwestern Conference on Fluid Mechanics, edited by A. M. KUETHE. Ann Arbor: The University of Michigan Press, 1957. 388 pp. $8.00.

This volume contains 26 of the 32 papers presented at the 5th Midwestern Conference, held at the University of Michigan in April 1957. The papers range over many different branches of fluid mechanics, and it is not possible to review the volume as a whole. Some of the papers that I was able to assess seemed to me to be good, although some others did not.

Although no specific review is called for, it may be useful to notice the volume as another example of the growing practice of publishing the proceedings of conferences of all kinds. There is an obvious case for publishing the proceedings (or some part of them) of a deliberative conference or symposium which has performed a job of work and has done something which the participants could not have done without meeting together. One might also argue in favour of publishing the proceedings of a non-deliberative conference on a restricted and important subject, on the grounds that even if the papers presented could be published in the regular journals it would be useful to have them all in one volume. But what about a conference at which, as at that under notice, the range of subject matter is enormous and the organizers presumably accept for publication whatever papers (within certain wide limits) are offered?

The purpose of a conference is undoubtedly to enable people to confer, and this it usually does well. Scientists and engineers are gregarious folk, as well as being internationally minded, and in these days of generous support from various international scientific unions and national agencies people are willing to go anywhere to talk about anything. Everyone has a good time, the equivalent of a vast amount of communication by paper is accomplished, and people have their outlooks enlarged by personal contact with a few leading spirits. This is fine, and as it should be. But why is the legitimate business of a conference confused with the matter of providing a written record of individual contributions to research? What a man says in a talk to a conference audience is not likely to be suitable for publication in exactly the same form, at any rate not if it is a good talk; the needs of the talk and the written record are different, not only in choice of wording but also in structure of the account. A talk is an end in itself, and it would be a pity if the prospect of publication of conference proceedings led speakers to prepare their talks with the needs of the written record in mind. The organizers of the Heat Transfer and Fluid Mechanics Institute (California) have the right idea; they issue preprints of papers to be read at the meetings and state that "the preprints are made available for discussion only and not as contributions in final form to the permanent literature. The authors are free to submit their papers for publication to any journal of their choice." However, since these preprints are reproduced and sold in book form by a regular publisher, the statement does not correspond completely with practice.

What puzzles me is that the practice of publishing in book form an almost random collection of papers read at a conference is becoming so common. On the face of it, it seems to suffer from the serious practical disadvantage that it does not serve the immediate interests of any of the parties concerned. The publishers have on their hands a volume which is unlikely to sell outside libraries and is hardly an economic proposition without a subsidy; the editors have the thankless task of cajoling a large number of authors into working to a schedule and then of imposing some kind of uniformity on the type-scripts; and the speakers at the conference are obliged to prepare a complete written account of their talk, even though their talk may be none the better, and may even be a little the worse, for the existence of a straight-jacket in the form of a detailed type-script complete with equations; moreover, their work (if not already published elsewhere in slightly different form) then appears, after an unpredictable delay, in an irregular publication of the kind that tends to become inaccessible. Someone in a position of influence must think it is a good idea. Or are the organizers of conferences simply being too conscientious about producing some tangible evidence of their work?

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